Geodesy 1 (GED203)

Lecture No: 3

CURVES ON ELLIPSOID

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Review of Previous Lecture

- SHAPES THAT APPROXIMATES THE EARTH'S SHAPE
- ELLIPSOID GEOMETRY
- LINEAR AND ANGULAR PARAMETERS OF ELLIPSOID
- ELLIPSOID AS REFERENCE FRAME
- GEODETIC (ELLIPSOIDAL) COORDINATES
- DIFFERENT TYPES OF LATITUDES
- RELATION BETWEEN TYPES OF LATITUDES
- **RELATION BETWEEN TOPOGRAPHY, ELLIPSOID, AND GEOID**
- BEST FITTING ELLIPSOID

Where are we till now?

please keep up

Overview of today's lecture

COMMON CURVES ON SURFACE ELLIPSOID

NORMAL SECTION

GEODESIC LINE CURVE

RADII OF CURVATURE

LENGTHS OF DIFFERENT TYPES OF ARCS

SUMMARY

Expected Learning Outcomes

- Recognizing and describing the different types of curves that can be found on the surface of an ellipsoid, including normal sections, geodesic lines, curve radii of curvature, and lengths of different types of arcs.
- Identifying the characteristics and properties of each type of curve, such as how they relate to the shape of the ellipsoid and their geometric properties.
- Analyzing and interpreting diagrams, equations, or examples related to the types of curves on an ellipsoid.
- Formulating questions or hypotheses about the behavior of curves on an ellipsoid and developing strategies to explore or investigate them.
- Demonstrating critical thinking skills by evaluating and comparing the different types of curves on an ellipsoid and their significance in different contexts.
- Applying problem-solving techniques to analyze and solve problems or exercises related to the types of curves on an ellipsoid.

What are the Types of Curves connecting any Two Points On Ellipsoid?

Curves on the Surface of an Ellipsoid

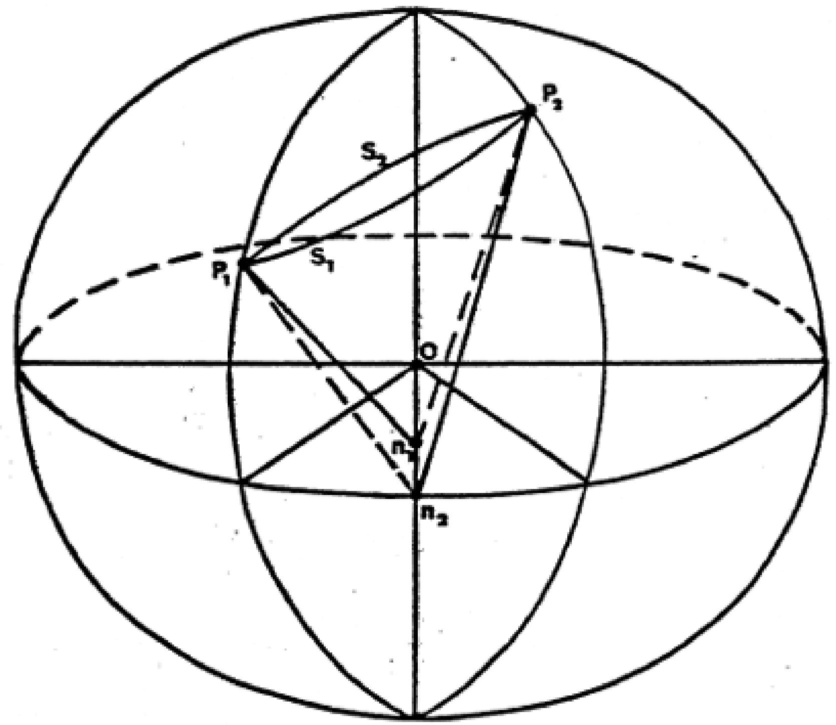
- There are ***two*** principal curves on the surface of an ellipsoid that are of special interest in geometric geodesy.

Normal section

Geodesic

Geodetic Arc Distances – Normal Section

- It is a plane curve created by intersecting a plane containing the normal to the ellipsoid (a normal section plane) with the surface of the ellipsoid.
- The line of intersection of a normal plane (at a point P) and the surface of the ellipsoid.
- Consider two points on the surface of an ellipsoid (P_1 and P_2) which are on different meridians and are at different latitudes.
- The normal section from P_1 to P_2 (*direct normal section*), is not coincident with the normal section from P_2 to P_1 (*inverse normal section*).



Reciprocal normal sections

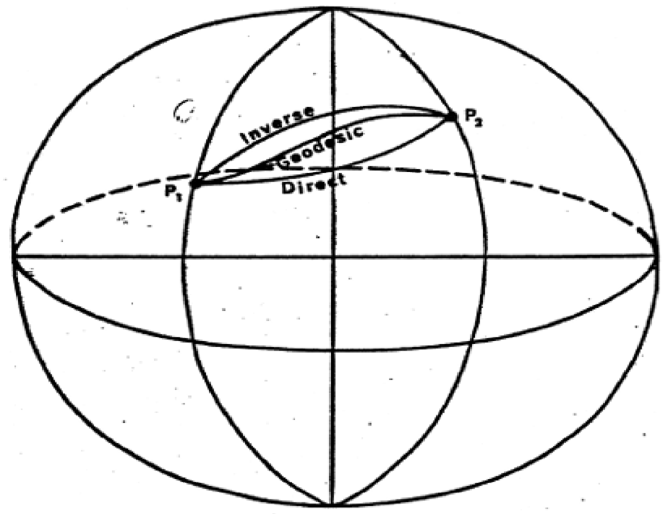
Direct and inverse geodetic problems will be explained in Lecture 6

Geodetic Arc Distances – Normal Section

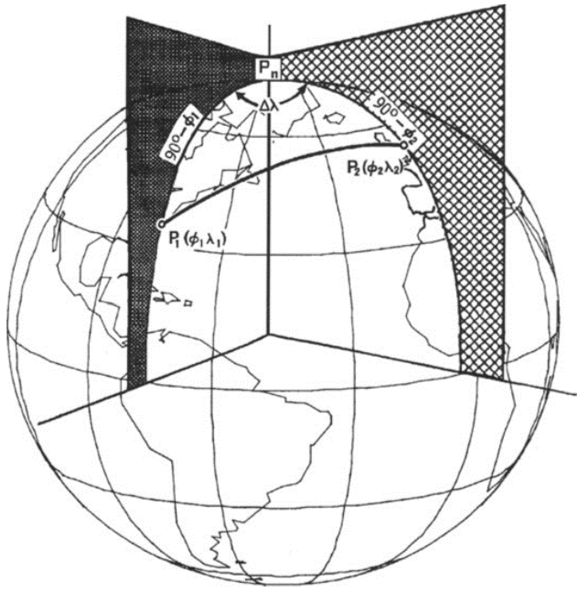
- It is *similar* to a geodesic, except that it is always a *plane curve*.
- It is *different* from a geodesic in that two normal sections exist between any two points, except in the cases of the meridians and the equator.
- The normal section does not give a unique line between two points.

Geodetic Arc Distances – Geodesic

- The geodesic, or geodetic line:- is the unique surface curve between any two points on the surface of an ellipsoid.
- At every point along the geodesic, the principal radius of curvature vector is coincident with the ellipsoidal normal.
- The geodesic between two points P_1, P_2, is the shortest surface distance between these two points.
- Any segment of a meridian or the equator is a geodesic.

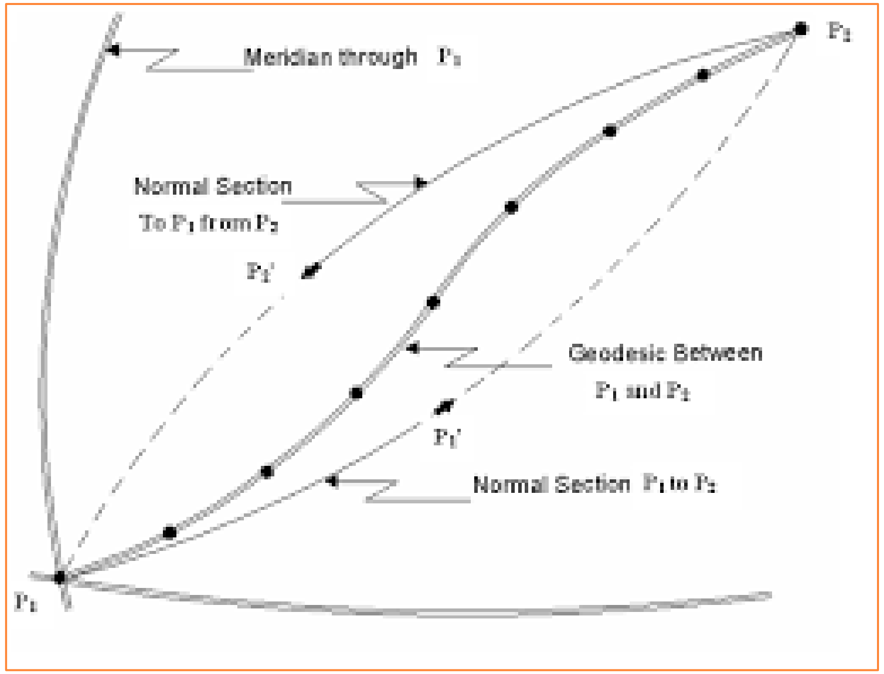


Geodesic

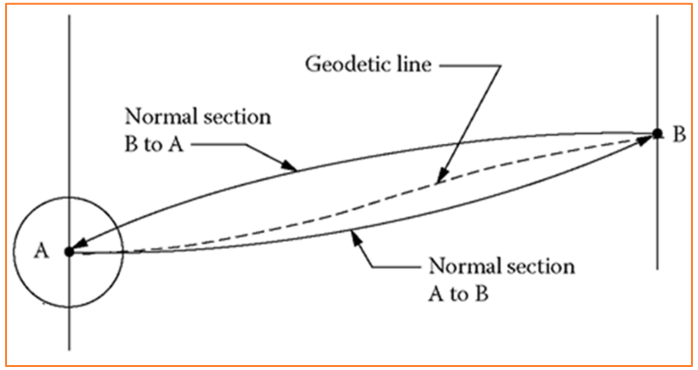


GEODETIC ARC DISTANCES – GEODESIC

- It is a curve on a surface where at each point of the curve the principal normal of the curve coincides with the normal to the surface of the ellipsoid at this point.
- It lies between two plane curves “normal sections” and has a double curvature.



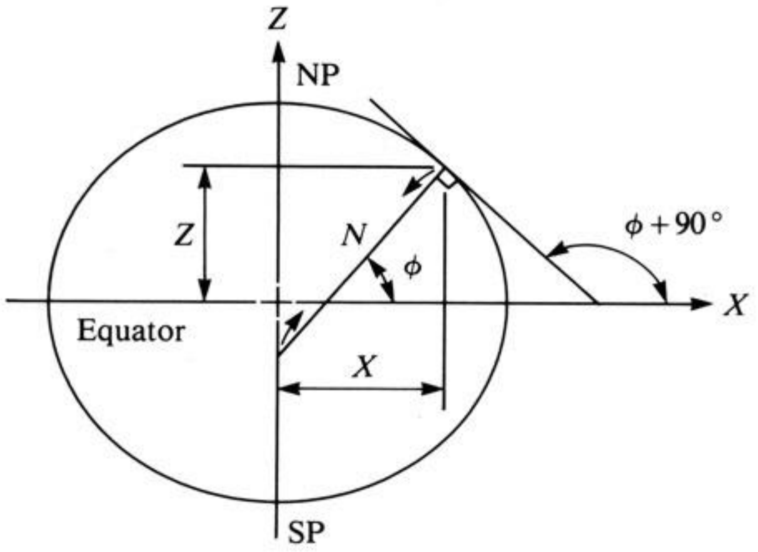
- It divides the angle of intersection at each vertex by $\frac{2}{1}$.
- The vertical plane on geodesic always contains the normal to ellipsoid.



Radii of Curvature On Ellipsoid

X and Z Coordinates on Ellipsoid

- On an ellipse, X and Z coordinates are represented as a function of geodetic latitude φ.
- According to the shown figure, their equations are: -



$$X = \frac{a\cos\varphi}{(1 - e^2 sin^2\varphi)^{0.5}} \quad (1)$$

$$Z = \frac{a(1-e^2)\sin\varphi}{(1 - e^2 sin^2\varphi)^{0.5}} \quad (2)$$

(1) Radius of Curvature in Prime Vertical Direction

- The radius of the circle that lies in the meridian plane and passes through the point of interest.

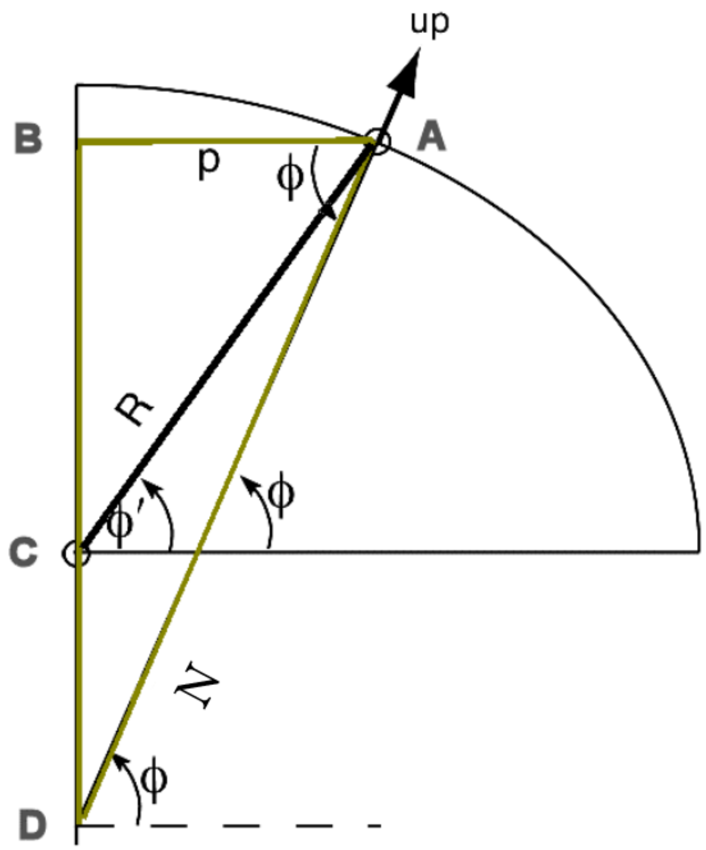
$$N = \frac{X}{\cos\varphi} \qquad (3)$$

Substitute by value of X from Eq. 1: -

$$N = \frac{a}{\sqrt{1-e^2 sin^2\varphi}} \qquad (4)$$

Eq. 4 represents the radius of curvature in the prime vertical.

What is the variable parameter in Eq. 4?



(1) Radius of Curvature in Prime Vertical Direction

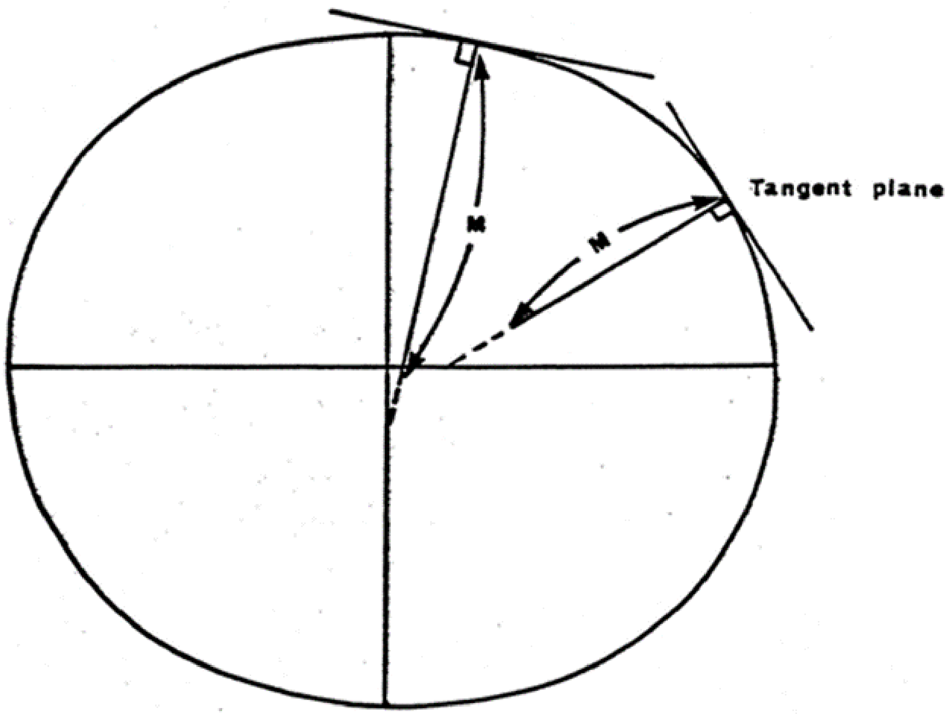
What is the variable parameter in Eq. 4?

- The only variable parameter is the "Geodetic latitude", therefore the range of values of N can be expected as follows: -
- $N = a$ ……………… At equator $\varphi = 0$. ($\sin 0 = 0$)
- $N = \frac{a}{(1-e^2)}$ ……….. At pole $\varphi = 90$. ($\sin 90 = 1$)

N is directly proportional to geodetic latitude.

(2) Radius of Curvature in Meridian Direction

- The radius of a circle that is tangent to the ellipsoid at the latitude and has the same curvature as the ellipsoid in the north-south direction there.
- The radius used for the latitude change to North distance.



(2) Radius of Curvature in Meridian Direction

- Consider a meridian section of an ellipsoid of rotation is given by: -

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (5)$$

- The radius of curvature of this curve, at any point P, is given by: -

$$M = \frac{\left(1+\left(\frac{dz}{dx}\right)^2\right)^{3/2}}{\frac{d^2z}{d^2x}} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (6)$$

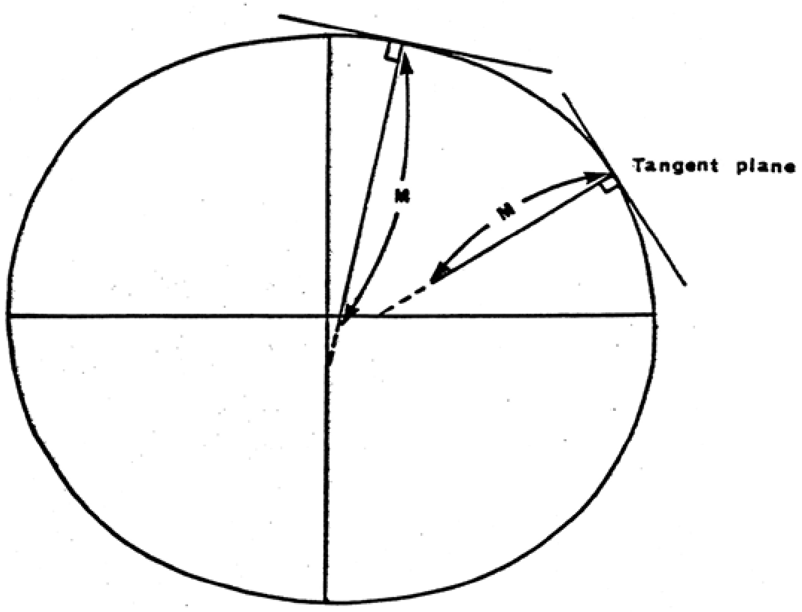
- In the ce.se of a meridian ellipse,

$$\frac{dz}{dx} = -\frac{x}{z}.\frac{b^2}{a^2} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (7)$$

From Eqs. 1 and 2, substitute in Eq.

$$M = \frac{a(1-e^2)}{(1-e^2 sin^2\varphi)^{\frac{3}{2}}} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (8)$$

Eq. 8 represents the radius of curvature in the meridian direction.



What is the variable parameter in Eq. 8?

(2) Radius of Curvature in Meridian Direction

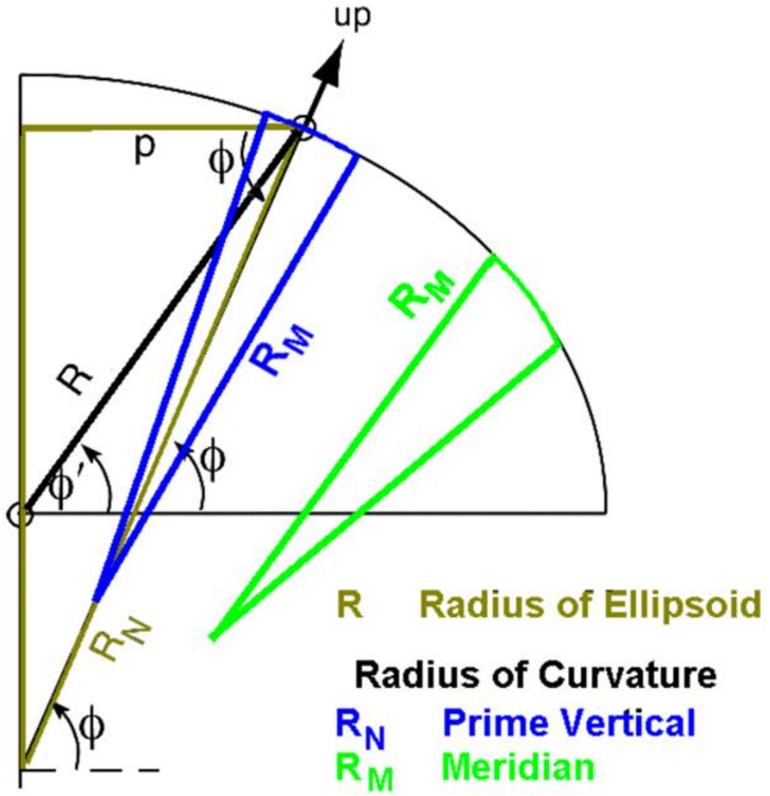
What is the variable parameter in Eq. 8?

- The only variable parameter is the "Geodetic latitude", therefore the range of values of M can be expected as follows: -
- $M = a(1 - e^2)$ ……………… At equator $\varphi = 0$. ($\sin 0 = 0$)
- $M = \frac{a}{(1-e^2)}$ ………………… At pole $\varphi = 90$. ($\sin 90 = 1$)

M is directly proportional to geodetic latitude.

TAKE HOME ASSIGNMENT (1)

- **Compare between the values of M and N using a range of latitude from 0 to 90 degrees with a step of 15 degrees.** Please consider all computations are made on Helmert1906 (a = 6378.2 km, $f = \frac{1}{298.3}$).
- **Draw a line plot to show the results. Feel free to use your custom visualization.**
- **Comment on your results to show their relationship (e.g., each of which is larger!).**

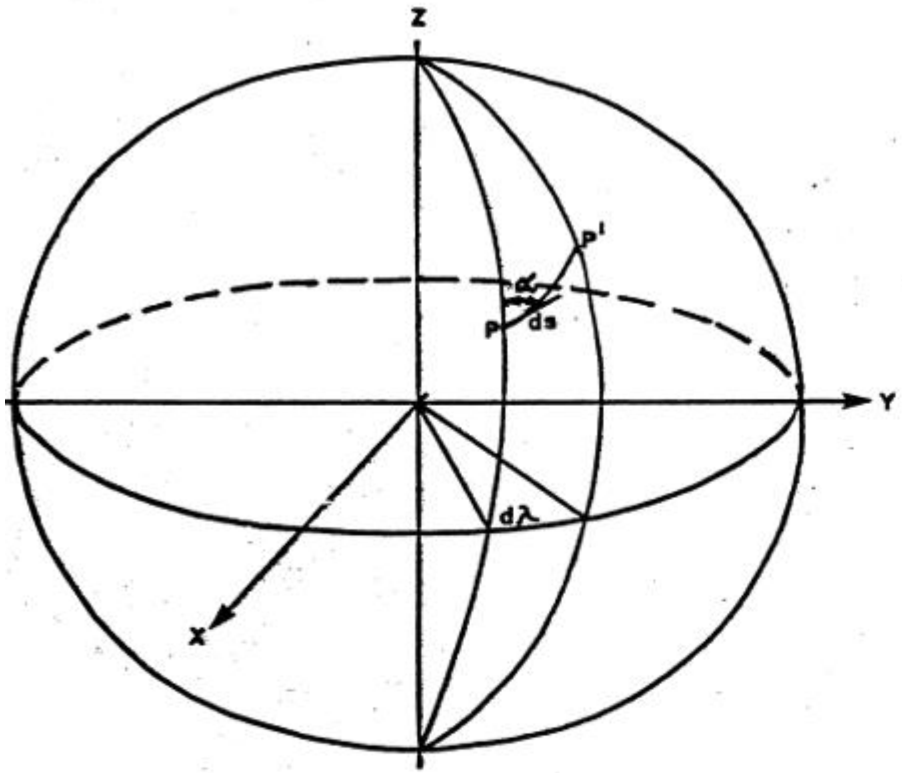


(3) Radius of Curvature at any Azimuth

- In some instances, geodetic computations require the radius of curvature in a plane other than the principal ones.
- The normal section in some azimuth α has a radius of curvature at any point P designated by R_α •

$$R_\alpha = \frac{MN}{M \sin^2\alpha + N \cos^2\alpha} \qquad (9)$$

- Also called Euler's radius of curvature.



Normal section at any azimuth α

(4) Mean Radius of Curvature of Ellipsoid

- An important quantity that is used very often in geometric geodetic computations is the Gaussian Mean radius of curvature, which is given by: -

$$R_m = \sqrt{MN} \qquad (10)$$

- In many instances, the mean radius is sufficiently accurate for position computations.

Lengths of Different Types of Arcs

(1) Meridian Arc Length

- If two points P_1 and P_2 lie on the same meridian then, the distance d_s between them is a part of meridian which is given by:-

$$d_s = M.d\varphi \quad (11)$$

thus,

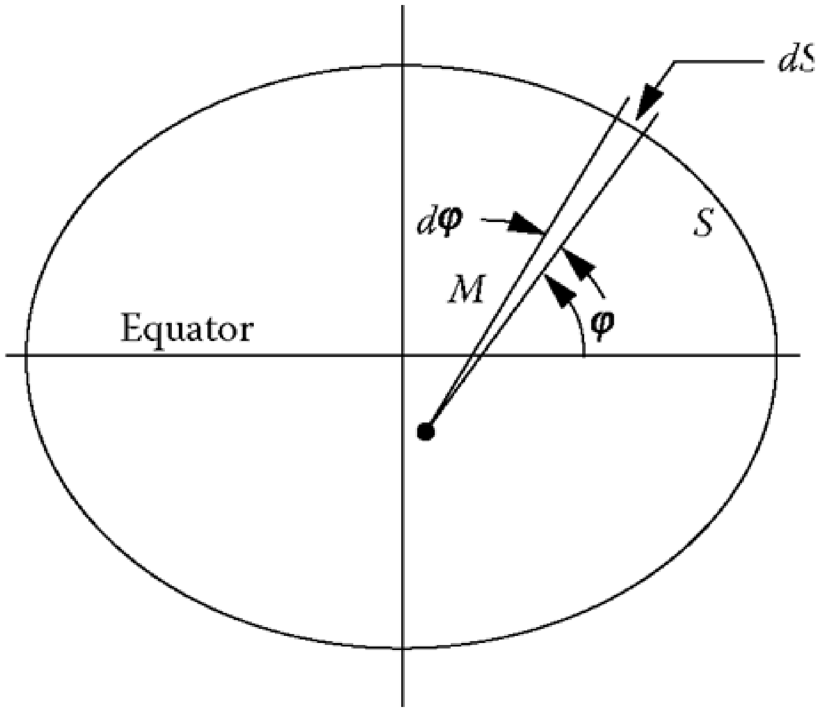
$$S = \int_{\varphi_1}^{\varphi_2} M.d\varphi \quad (12)$$

For distances up to 400 km,

$$S = M.\Delta\varphi\ (1 + \frac{1}{8}\ e^2\Delta\varphi^2 \cos 2\varphi) \quad (13)$$

If distance is less than 45 km: -

$$S = M.\Delta\varphi \quad (14)$$



Meridian cross section

(2) Parallel Arc Length

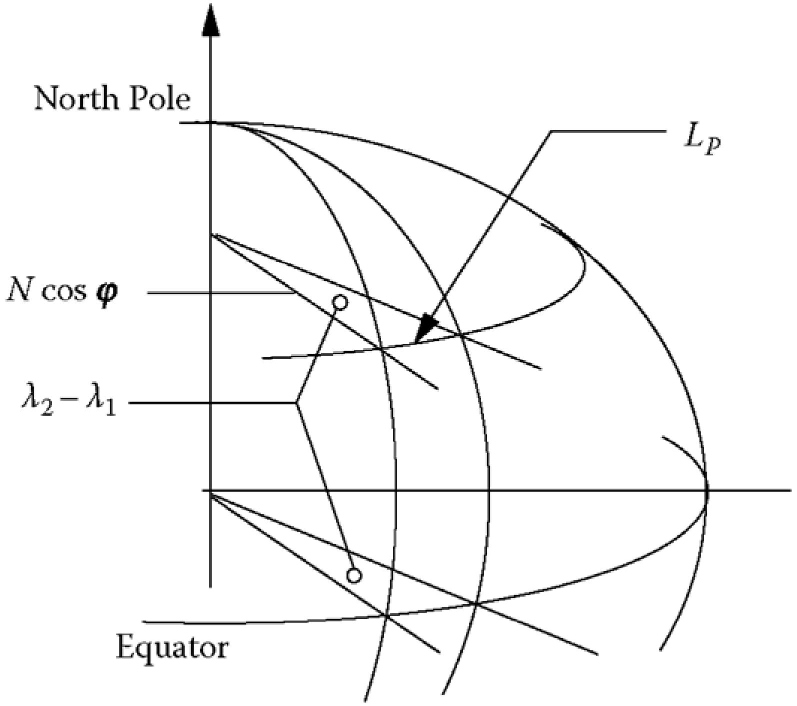
- If two points P_1 and P_2 lie on the same meridian then, the distance d between them is a part of parallel of latitude which is given by:-

$$L_p = \rho \times \Delta \lambda^r \quad \text{(15)}$$

Nevertheless ρ = N cos φ.

Therefore,

$$L_p = N \cos \varphi \times \Delta \lambda^r \quad \text{(16)}$$



Open Discussion

Table 5.4 Variations in meridian and parallel arc lengths with latitude *B* (GRS80 Ellipsoid)

	Length of a meridian arc (m)			Length of a parallel arc (m)		
B	$\Delta B = 1^{\circ}$	$\Delta B = 1'$	$\Delta B = 1''$	$l = 1^{\circ}$	$l = 1'$	$l = 1''$
0°	110,574	1,842.91	30.715	111 321	1,855.36	30.923
15°	110,653	1,844.15	30.736	107 552	1,792.54	29.876
30°	110,861	1,847.54	30.792	96 488	1,608.13	26.802
45°	111,141	1,852.20	30.870	78 848	1,341.14	21.902
60°	111,421	1,856.87	30.948	55 801	930.02	15.500
75°	111,623	1,860.30	31.005	28 902	481.71	8.028
90°	111,694	1,861.57	31.026	0.000	0.000	0.000

How do these Computations differ if earth is modeled as a sphere?

Radius of Sphere

- Unique Radius R

Therefore,

- length of meridian arc is:

$$S = R\,\Delta\varphi^{r} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (17)$$

- Length of parallel arc is:

$$L_p = R\cos\varphi \times \Delta\lambda^{r} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (18)$$

Online Resources

- Computation of meridian arc (http://www.in-dubio-pro-geo.de/?file=ellip/marc0&english=1)
- Calculate distance, bearing and more between Latitude/Longitude points (https://www.movable-type.co.uk/scripts/latlong.html).
- ***Geopy*** is a Python library that simplifies the calculation of geographic distances between two points (https://pypi.org/project/geopy/)

LET'S SUMMARIZE

NEXT TUESDAY

Lecture 4 - Spherical and Ellipsoidal Triangles

Be Prepared

Time of Quiz 1

GEODESY

THANK YOU

End of Presentation